Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem- 1st Regular Examination January 2011

Subject code: 110008
Subject Name: Mathematics I

Date:11/ 01 /2011 **Time: 10.30 am - 01.00 pm**
Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as Directed

i) Can the Rolle's theorem for $f(x)=|x|, x \in [-1,1]$ applied? **02**

ii) If $\frac{1}{2}-\frac{x^2}{24}<\frac{1-\cos x}{x^2}<\frac{1}{2}$ hold for values of x close to zero, find $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ **02**

iii) Find the absolute maximum and minimum value of $f(x)=x^{2/3}$ on the interval [-2,3] **02**

iv) Find c of the Mean Value theorem for $f(x)=\log x \;; x\in[1,e]$ **02**

v) Find the sum of the series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ **02**

vi) Prove that $\int_1^3 (x^2-x)\,dx \geq 0$ **02**

vii) Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point(4,-5) if $f(x,y)=x^2+3xy+y-1$ **02**

Q.2 (a) i) Find parametric equation of the line joining (-3,2) and (2,-1) **01**

ii) Expand $\sin(x+h)(y+k)$ by Taylor's Series **02**

iii) If $u=\cos ec^{-1}\left(\frac{x+y}{x^2+y^2}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$ **02**

iv) Obtain $curl\vec{F}$ at the point (2,0,3), $\vec{F}=ze^{2xy}\overline{i}+2xy\cos y\,\overline{j}+(x+2y)\overline{k}$ **02**

(b) i) Trace the curve $y=2x+\frac{x^2}{2}-\frac{x^3}{3}$ **04**

ii) Verify Cauchy's Mean value theorem for $2x^3$ and $x^6, x\in[a,b], a>0$ **03**

OR

(b) i) Trace the curve $r=a(1+\cos\theta); a>0$ **04**

ii) Find the Taylor's series generated by $f(x)=\frac{1}{x}$ at a=2.Where, if anywhere does the series converge to $\frac{1}{x}$? **03**

Q.3 (a) i) Does the sequence whose n^{th} term is $a_n=\left(\frac{n+1}{n-1}\right)^n$ converge? if so, find $\lim_{n\to\infty} a_n$ **04**

		ii)	If $a_n = \begin{cases} \frac{n}{2^n}, n\ odd \\ \frac{1}{2^n}, n\ even \end{cases}$ does $\sum a_n$ converges?	**03**
	(b)	**i)**	The region between the curve $y=\sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.	**04**
		ii)	Prove that $\beta(m,n)=\beta(m+1,n)+\beta(m,n+1)$	**03**
			OR	
Q.3	**(a)**	**i)**	For the function $f(x)=x+x^2$ find a formula for the upper sum obtained by dividing the interval $[0,1]$ into n equal subintervals, then take a limit of these sums as $n \to \infty$ to calculate the area under the curve over$[0,1]$	**04**
		ii)	Test the Convergence for the series $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$	**03**
	(b)	**i)**	Evaluate $\int_0^1 x^m (\log x)^n dx$, where n is a positive integer and m>1	**04**
		ii)	Find the length of the curve $y=x^{3/2}, 0 \le x \le 1$	**03**
Q.4	**(a)**		Find the greatest and smallest values that the function $f(x,y)=xy$ takes on the ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$	**05**
	(b)		If $u=f(r)$ where $r^2=x^2+y^2$, prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$	**05**
	(c)		Evaluate $\iint_R x^2 dA$, where R is the region in the first quadrant bounded by the hyperbola $xy=16$ and the lines $y=x, y=0$ and $x=8$.	**04**
			OR	
Q.4	**(a)**		Find a point within a triangle such that the sum of the squares of its distances from the three vertices is a minimum	**05**
	(b)		Find the point on the plane $x+2y+3z=13$ closest to the point (1,1,1)	**05**
	(c)		Evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$ by changing the order of integration	**04**
Q.5	**(a)**		Integrate $f(x,y)=x^2+y^2$ over the triangular region with vertices (0,0),(1,0) and (0,1)	**05**
	(b)		Find the directional derivative of $div\vec{F}$ at (2,2,1) in the direction of normal to the sphere $x^2+y^2+z^2=9$,where $\vec{F}=x^2z\bar{i}+xy^2\bar{j}+yz^2\bar{k}$	**05**
	(c)		Find the area of the surface cut from the bottom of the paraboloid $x^2+y^2-z=0$ by the plane $z=4$	**04**
			OR	
Q.5	**(a)**		Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4$ and $z=0$	**05**
	(b)		Find the angle between the surface $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2)	**05**
	(c)		Verify Stoke's theorem for $\vec{F}=y\bar{i}+z\bar{j}+x\bar{k}$,where S is the upper half of the sphere $x^2+y^2+z^2=1$ and C is its boundary	**04**

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