Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1st / 2nd EXAMINATION– WINTER 2015

Subject Code: 110008 **Date:28/12/2015**

Subject Name: Mathematics – 1

Time: 10:30am to 01:30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks**

Q.1 **(a)** Using L'hospital rule, evaluate following limits:

1. $\lim_{x\to 0}\dfrac{\tan x-\sin x}{x^2}$ **03**
2. $\lim_{x\to 1}(1-x)\tan\left(\dfrac{\pi x}{2}\right)$ **02**
3. $\lim_{x\to 0} x^{n-1}\ln x;\ n>1$ **03**

(b) Test the convergence or divergence of following series:

1. $\sum\dfrac{1}{n!}$ **03**
2. $\sum_{n=1}^{\infty}\dfrac{1}{\left(1+\dfrac{1}{n}\right)^{n^2}}$ **03**

Q.2 **(a)** If $u=x^2y+y^2z+z^2x$, then prove that $\left(\dfrac{\partial}{\partial x}+\dfrac{\partial}{\partial y}+\dfrac{\partial}{\partial z}\right)^2 u=3\left(u_{xx}+u_{yy}+u_{zz}\right)$ **07**

(b) State Euler's theorem for homogeneous function and verify it for $u=\sqrt{x}+\sqrt{y}+\sqrt{z}$ by direct differentiation. **07**

Q.3 **(a)** Evaluate $\int_0^a\int_{\sqrt{ax}}^{a}\dfrac{y^2\,dy\,dx}{\sqrt{y^4-a^2x^2}}$ by changing the order to integration. **07**

(b)

1. Find the volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$ **05**
2. Using double integration find area of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ **02**

Q.4 **(a)** Verify Divergence theorem for $\bar{F}=\left(x^2-yz\right)i+\left(y^2-zx\right)j+\left(z^2-xy\right)k$ taken over the rectangular parallelepiped $0<x<a;\ 0<y<b;\ 0<z<c$ **06**

(b) Do as directed

1. Evaluate $\int_C(x+y)\,dx-x^2dy+(y+z)\,dz$ where C is $x^2=4y$, $z=x$, $0\le x\le 2$ **04**
2. Find the directional derivative of $f(x,y,z)=xy^2+yz^3$ at the point $(2,-1,1)$ in the direction normal to the surface $x\log z-y^2=-4$ at $(-1,2,1)$ **04**

Q.5 **(a)** The pressure P at any point (x, y, z) in the space $P = 400xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$ **05**

(b) Find the equation of tangent plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point $(2, 2, 1)$ **05**

(c) Find out linearization of $f(x, y, z) = x^2 + 2y^2 + 3z^2 + 6$ at $(1, 1, 1)$ **04**

Q.6 **(a)** If $a < b$ then prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ **05**

(b) Using Taylor's expansion theorem, expand $f(x) = e^x$ at $x = 0$ **05**

(c) Prove that $\int_1^{\infty} \frac{\cos x}{x^2}\, dx$ converges. **04**

Q.7 **(a)** Expand $e^x \cos y$ at $\left(1, \frac{\pi}{4}\right)$ **05**

(b) Test convergence of following series $\sum_{n=1}^{\infty} \left(\frac{n+1}{n+2}\right)^n x^n;\ x > 0$ **05**

(c) Find the point of inflexion on the curve $y = 4(x+3)^3$ **04**

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