Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – WINTER 2013

Subject Code: 110009 **Date: 17-12-2013**

Subject Name: **Maths-II**

Time: 10:30 am – 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Solve the following system of equations using Gauss - Jordan method. **05**

$$x-y+2z-w=-1,\quad 2x+y-2z-2w=-2$$
$$-x+2y-4z+w=1,\quad 3x-3w=-3$$

(b) Find the values of l, m and n if $A=\begin{bmatrix}0 & 2m & n\\ l & m & -n\\ l & -m & n\end{bmatrix}$ is orthogonal. **05**

(c) Find the rank of the matrix $\begin{bmatrix}2 & 1 & 3 & -1\\ 3 & -1 & 2 & 0\\ 1 & 3 & 4 & -2\\ 4 & -2 & 1 & 1\end{bmatrix}$ by reducing into normal form. **04**

Q.2 **(a)** Find the inverse of $\begin{bmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{bmatrix}$ **05**

(b) Examine the system **05**

$x-2y+z-3w=-3,\ -3x+y-z+2w=2,\ 4x+3y-3z+w=1$

for consistency and solve the system if consistent.

(c) Determine whether the vectors (3, 1, 4), (2, -3, 5), (5, -2, 9) and (1, 4, -1) span the vector space R^3. **04**

Q.3 **(a)** Find a basis for the subspace of R^3 spanned by the vectors (1, 0, 0), (0, 1, -1), (0, 4, -3) and (0, 2, 0). **05**

(b) Find a basis for the row space of $A=\begin{bmatrix}1 & 4 & 5 & 6 & 9\\ 3 & -2 & 1 & 4 & -1\\ -1 & 0 & -1 & -2 & -1\\ 2 & 3 & 5 & 7 & 8\end{bmatrix}$ consisting entirely the row vectors of A. **05**

(c) For what values of λ the vectors $\left(\lambda, \frac{-1}{2}, \frac{-1}{2}\right)$, $\left(\frac{-1}{2}, \lambda, \frac{-1}{2}\right)$ and $\left(\frac{-1}{2}, \frac{-1}{2}, \lambda\right)$ are linearly independent. **04**

Q.4 **(a)** Consider the basis S = $\{v_1, v_2\}$ of R^2, where $v_1=(1, 1)$ and $v_2=(2, 3)$. **05**

Let $T:R^2 \to P_2$ be the linear transformation such that $T(v_1)=2-3x+x^2$ and $T(v_2)=1-x^2$. Find the formula of $T(a, b)$.

(b) Verify dimension theorem for the linear transformation $T: R^4 \to R^3$ given by the formula $T(x, y, z, w) = (4x + y - 2z - 3w,\ 2x + y + z - 4w,\ 6x - 9z + 9w)$ **05**

(c) Find the matrix of the linear transformation $T: R^3 \to R^3$ defined by the formula $T(x, y, z) = (x + 2y + z,\ 2x - y,\ 2y + z)$ with respect to the basis $S = \{(1, 0, 1),\ (0, 1, 1),\ (0, 0, 1)\}$. **04**

Q.5 **(a)** Check whether the vectors $p = x$ and $q = x^2$ of P_2 are orthogonal relative to the inner product $< p, q > = \int_{-1}^{1} p(x)q(x)\,dx$ and if so, verify the Pythagorean Theorem. **05**

(b) Let R^3 have the inner product $\langle (x_1, x_2, x_3), (y_1, y_2, y_3) \rangle = x_1 y_1 + 2x_2 y_2 + 3x_3 y_3$. Use the Gram - Schmidt process to transform the set $S = \{(1, 1, 1),\ (1, 1, 0),\ (1, 0, 0)\}$ into an orthonormal set. **05**

(c) Let $S = \text{span}\{(0, 1, 0),\ (-4, 0, 3)\}$. Express $w = (1, 1, 1)$ in the form $w = w_1 + w_2$ where $w_1 \in S$ and $w_2 \in S^{\perp}$. **04**

Q.6 **(a)** Determine the algebraic and geometric multiplicity of each of the eigen values of $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ **05**

(b) Verify Cayley - Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and hence find A^4. **05**

(c) Find the non singular matrix P that diagonalizes $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ **04**

Q.7 **(a)** Find the change of variables that reduces the quadratic form $7x^2 + 5y^2 + 6z^2 - 4xz - 4yz$ to sum of squares. **05**

(b) Find the least squares solution of the linear system of equations $-2x + y = -2,\quad x + y = 4,\quad -x + y = 1,\quad 2x + y = 6.$ **05**

(c) Show that the linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (3x + y,\ 3x + 2y)$ is one to one and find $T^{-1}(x, y)$ **04**

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