Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1st / 2nd EXAMINATION– WINTER 2015

Subject Code: 110014 **Date:28/12/2015**

Subject Name: Calculus

Time: 10:30am to 01:30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks**

Q.1 **(a)** **(i)** Expand the polynomial f(x) = $x^5 + 2x^4 - x^2 + x + 1$ in power of x+1. **05**

(ii) Check whether the series $\sum \frac{1}{\sqrt{n}}$ is Convergent or Divergent. **02**

(b) **(i)** Test for the convergence for the series $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{4}{5!}+\ldots.$ **04**

(ii) Test for the convergence for the series $\sum \frac{(-1)^{n+1}}{\log(n+1)}$ **03**

Q.2 **(a)** **(i)** Trace the curve $y^2(a+x)=x^2(a-x)$, Where a > 0. **05**

(ii) Determine the concavity of y = 3+sinx on [0, π] **02**

(b) **(i)** Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}$ dx dy. **04**

(ii) For what values of x the power series $\sum \frac{x^n}{n!}$ Converge ? **03**

Q.3 **(a)** **(i)** Evaluate $\int_0^1 x^5 \sin^{-1} x\,dx$ **05**

(ii) Write down maclaurin series for sinx and cosx. **02**

(b) **(i)** Obtain reduction formulae for $\int_0^{\frac{\pi}{2}} \sin^n x\ dx, n \in N$. **04**

(ii) Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x\,dx$ **03**

Q.4 **(a)** **(i)** Change the order of integration to evaluate $\int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y} dy dx$. **05**

(ii) Use the Fundamental theorem to find $\frac{dy}{dx}$ if $y = \int_x^5 3t \sin t \, dt$ **02**

(b) **(i)** Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{x} \right)^{\frac{1}{x}}$. **04**

(ii) Find the area between parabolas $x^2 = 4ay$ and $y^2 = 4ax$ where a is constant. **03**

Q.5 **(a)** **(i)** Find the volume of the pyramid whose base is a square with side L and whose height is h. **05**

(ii) Evaluate $\int_1^\infty \frac{1}{1+t^2} dt$ **02**

(b) **(i)** Use cylindrical shells to find volume of the solid obtained by rotating about the y-axis the region between y =x and y = x^2. **04**

(ii) Find the volume of the solid obtained by rotating the region bounded by $y = x^3$,y = 8 and x = 0 about the y-axis. **03**

Q.6 **(a)** **(i)** If u(x, y) = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$ then prove that **05**

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = -\frac{\sin u \cos 2u}{4 \cos^3 u}.$$

(ii) Find the value of $\frac{\partial f}{\partial y}$ at the point (4, -5) for f(x, y) = $x^{2014} + 3xy + y - 2014$ **02**

(b) **(i)** If $u = x^4 y + y^2 z^3$ where $x = r s e^t$, $y = r s^2 e^{-t}$, and z = $r^2 s \sin t$, find the value of $\frac{\partial u}{\partial s}$ when $r = 1, s = 1, t = 0$. **04**

(ii) Sketch the level curves of the function f(x, y) = 6-3x-2y for the values k = 6,0,-6. **03**

Q.7 **(a)** **(i)** Find the local maximum and minimum values and saddle points of f(x, y) = $x^4 + y^4 - 4xy + 1$ **05**

(ii) Evaluate $\int_0^1 \int_0^{1-y} \int_0^2 dx dz dy$ **02**

(b) **(i)** Find the Tangent Plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point (1, 1, 3). **04**

(ii) Evaluate $\lim_{(x,y) \to (1,2)} \left(x^2 y^3 - x^3 y^2 + 3x + 2y \right)$ **03**