Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 110015 **Date: 05-01-2015**

Subject Name: Vector Calculus and Linear Algebra

Time: 10:30 am - 01:00 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (1) Solve the following system of equations (if possible)using Gauss eliminatin method. $2x+y-z=4,\quad x-y+2z=-2\quad -x+2y-z=2$ (14)

(2) If for two vectors $\bar{a}$ and $\bar{b}$, $|\bar{a}+\bar{b}|=|\bar{a}-\bar{b}|$ .Find angle between $\bar{a}$ and $\bar{b}$.

(3) Show that the linear operator $T:R^2\to R^2$ defined by the equation $w_1=2x+y$, $w_2=3x+4y$ is one-one and find $T^{-1}(w_{1,}w_2)$.

(4) Prove or disprove that the set $W=\{(a,2a,a+1)\mid a\in R\}$ is or not vector-subspace of R^3.

(5) Find two vectors in R^2 with Euclidian Norm whose inner product with (-3,1) is zero.

(6) Find the eigenvalue and eigenvector of the matrix A= $\begin{bmatrix}1 & -2\\ -5 & 4\end{bmatrix}$.

(7) Let S =$\{v_{1,}v_2,v_3\}$ be the basis for R^3 in the $v_1(1,2,1)$ $v_2(2,9,0)$ $v_3(3,3,4)$ find the coordinate vector of V =(5,-1,9) .

Q.2 (a) (1) Find A^{-1} by Gauss Jordan method where $A=\begin{bmatrix}2 & 6 & 6\\ 2 & 7 & 6\\ 2 & 7 & 7\end{bmatrix}$. (3)

(2) Solve following equation by Cramer's rule. (4)
$x+2z=6$, $-3x+4y+6z=30$, $-x-2y+3z=8$.

(b) (1) Investigate for what values of λ and μ the equations (4)
$x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ .have (i)no solution (ii)a unique solution (iii)an infinite number of solution.

(2) Reduce the following matrix into reduced raw echelon form (3)

$$A=\begin{bmatrix}1 & 2 & 3 & -1\\ -2 & -1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & -1\end{bmatrix}$$

Q.3 (1) Determine which sets are vector space or not under the giver operation. The set all triple of real no $\{x,y,z\}$ with the operations (5)

$(x,y,z)+(x',y'.z')=(x+x',y+y',z+z')$ and $(x,y,z)=(kx,y,z)$

(2) Define basis and Determine the dimension and basis for the solution space of

the system $3x + y + z + w = 0 \quad 5x - y + z - w = 0$ (4)

(3) State Dimension theorem and verify that for the given matrix

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$ (5)

Q.4 (1) Consider the basis S = $\{v_{1,} v_2, v_3\}$ for R^3, $v_1(1,2,1)$ $v_2(2,9,0)$ $v_3(3,3,4)$ and let $T: R^3 \rightarrow R^2$ be the linear transformation such that $T(v_1) = (1,0)$ $T(v_2) = (-1,1)$, $T(v_3) = (0,1)$. Find a formula for $T(x_1, x_2, x_3)$ and use the form formula to find $T(7,13, 7)$. (5)

(2) Determinew whether the given vectors $v_1 = (2,-1,3)$ $v_2 = (4,1,2)$ $v_3 = (8,-1,8)$ span R^3. (3)

(3)Let $v_1 = (1,-1,0)$ $v_2 = (0,1,-1)$ $v_3 = (0,0,1)$ be elements in R^3.Verify the set of vectors (v_1, v_2, v_3) is linearly independent or linearly dependent. (3)

(4)Find the basis for the raw space for the matrix A

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$ (3)

Q.5 (1) Find the eigenvalue and eigenvector for the matrix (3)

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

(2) find a matrix P that diagonalizes A and determine $p^{-1}AP$ (5)

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(3) Verify Caley-Hamilton theorem for the matrix. (3)

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(4) Reduce the quadratic form Q(x,y,z)= $3x^2 + 3z^2 + 4xy + 8xz + 8yz$ into canonical form using linear transformaion. (3)

Q.6 (1) Let V be an inner product space and let u,v $\in$ V Prove that

$$\langle u, v \rangle = \frac{1}{4}\|u + v\|^2 - \frac{1}{4}\|u - v\|^2$$ (2)

(2) Let u=(u_1, u_2) and v=(v_1, v_2) be a vector in R^2 Verify that the weighted Euclidean inner product on R^2 $\langle u, v \rangle = 3u_1v_1 + 2u_2v_2$. (4)

(3) Let R^3 have the Euclidean inner Product. Use the Gram-Schmidt process to transform the bases $\{u_1, u_2, u_3\}$ into orthogonal bases $u_1 = (1,1,1)$

$u_2 = (-1,1,0)$ $u_3 = (1,2,1)$ (4)

(4) Find a least square solution of the inconsistent system Ax=b for

$$A=\begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \ b=\begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$ (4)

Q.7 (1) If $\phi = 3x^2y - y^3z^2$, find the grad ϕ at the point (1,-2,-1). (2)

(2) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at the point (2,-1,2) in the Direction (2,3,6). (4)

(3) Using Green's theorem, evaluate $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ Where c is the Boundary of the region boubded by $y^2 = x$ and $x^2 = y$. (4)

(4) Evaluate $\iint_s 6xy\,ds$ where s is the portion of the plane $x + y + z = 1$ that lies in front of Yz plane (4)

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