Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd EXAMINATION (New Syllabus) – WINTER 2014

Subject Code: 2110014 **Date: 29-12-2014**
Subject Name: Calculus
Time: 10:30 am - 01:30 pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question**

(a) Answer the following questions with most appropriate answer **07**

1. The maximum value of $y = 3\cos 2x$ is ____

(a) 1 (b) 2 (c) -1 (d) 3

2. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then dy/dx ____

(a) $\sqrt{y/a}$ (b) $\sqrt{x/a}$ (c) $-\sqrt{y/x}$ (d) $-\sqrt{y/a}$

3. The $(a - x)y^2 = x^2(a + x)$ is symmetric about

(a) X-axis (b) Y-axis (c) both X and Y axis (d) line $Y = X$

4. Tangents at origin to the curve $y^2(a + x) = x^2(3a - x)$ is ____

(a) $\pm\sqrt{3}\,x$ (b) 1 (c) $\pm\sqrt{2}\,x$ (d) none of these

5. The parametric equations of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ $(a \neq 0)$ are

(a) $x = a\cos\theta,\ y = a\sin\theta$ (b) $x = a^3\cos\theta,\ y = a^3\sin\theta$

(c) $x = a\cos^3\theta,\ y = a\sin^3\theta$ (d) x=1, y=0

6. The area bounded by the curve $y = x^3$, x-axis and two ordinates $x = 1$ to $x = 2$ equal to

(a) 15/2 (b) 15/3 (c) 15/5 (d) 15/4

7. $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x^2 - 4} =$ ___

(a) 7/4 (b) 3/4 (c) 2 (d) -3/4

(b) Answer the following questions with most appropriate answer **07**

1. The slope of the tangent to the curve $y = xe^x$ at $(0,0)$ is___

(a) 0 (b) 1 (c) -1 (d) 4

2. The point of inflection of $\dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x + 14$ is ____

(a) 1/2 (b) -1/2 (c) -1 (d) -2

3. Using the matrix method, the solution of $x + y = 2,\ 4x + y = 6,$

(a) $\left(\dfrac{-4}{3}, \dfrac{2}{3}\right)$ (b) $\left(\dfrac{4}{3}, \dfrac{-2}{3}\right)$ (c) $\left(\dfrac{4}{3}, \dfrac{2}{3}\right)$ (d) (1,2)

4. $\int e^{2x} \cos 3x\,dx =$ ______ $+ c$

(a) $\dfrac{e^{2x}}{13}(2\cos 3x + 3\sin 3x)$ (b) $\dfrac{e^{2x}}{13}(3\cos 3x + 2\sin 3x)$

(c) $\dfrac{e^{2x}}{13}(2\cos 3x - 3\sin 3x)$ (d) none of these

5. If $y = \ln(\sqrt{2}\,x)$ the derivative of the function y with respect to x is ____

(a) 0 (b) -1/2x (c) 1/2x (d) 1/x

6. $f(x) = x^2 + 4x + 5$ has the minimum value ____

(a) 0 (b) 1 (c) -1 (d) 2

7. A curve which passes through the origin and has the slope -1/3 is given by

(a) $x+3y-1=0$ (b) $x+3y=0$ (c) $x-3y=0$ (d) none of these

OR

Q:1 (a) Answer the given MCQ. **07**

1. F(x) is strictly increasing function on R then______

A) f'(x) = 0 for all x

B) f'(x) > 0 for all x

C) f'(x) < 0 for all x

D) none of these

2. F(x) is strictly decreasing function on R then______

A) f'(x) = 0 for all x

B) f'(x) > 0 for all x

C) f'(x) < 0 for all x

D) none of these

3. dy/dx = ky, k > 0 is the deferential equation for_______

A) Population Model C) Cooling Model

B) Mixing problem model D) none of these

4. dy/dx = x then y = _______

(A) $y = x^2$ B) $y = \frac{x^2}{2} + c$ C) $y = x$ D) $non\ of\ these$

5. Curve of $y = x^2 + 3$ is______

A) Symmetric with respect to x axis. B) Symmetric with respect to y axis.

C) Symmetric with respect to origin. D) none of these

6. r = a cosθ is_____

A) $Line$ C) $circle$

B) $laminiscate$ D) None of these

7. $\int_0^1 x^2 dx$ is ______

A) $area\ under\ a\ line$ C) $area\ under\ parabola$

B) $area\ under\ circle$ D) none of these

(b) Answer the given MCQ. **07**

1. $\int_a^{-a} f(x)dx = 0\ if$ ____________

A) f is an odd function C) f is an even function

B) f is neither odd nor even function D) none of these

2. $z = x^2 + y^2$ is ________

A) Cone C) $sphere$

B) $paraboloid$ D) $none\ of\ there$

3. $\lim_{(x,y)\to(0,0)} \frac{x^2-yx}{x+y} =$ ________

A) 2 B) 1 C) 0 D) -1

4. If $z = x^2 - y^2$ then $\frac{\partial z}{\partial x} =$ ___

A) 2y C) 2z

B) 0 D) none of these

5. Equation of tangent plane of z = x at (2,0,2) is

A) $z = x$ C) $x + z = 0$

B) $x + y + z = 2$ D) $None\ oFthese$

6. If $z = x^2 + y^2 + 3$ minimum value z is ______

A) 3 C) 0

B) ∞ D) *none of these*

7. Y = sin2x is increasing in interval ________

A) $(0,\pi)$ C) $(0,\frac{\pi}{2})$

B) $(0,\frac{\pi}{4})$ D) None of these

Q.2 **(a)** Test the convergence of $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+....$ **03**

(b) Prove that $1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+....$ converges and find it's sum. **04**

(c) State Taylor's series for one variable and hence find $\sqrt{36.12}$ **07**

Q.3 **(a)** Find $\frac{\partial w}{\partial r}, \frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$ where $x = \frac{r}{s}$, $y = r^2 + \ln s$, $z = 2r$ **03**

(b) Find $\frac{dy}{dx}$ when $(\cos x)^y = (\sin y)^x$ **04**

(c) If $u = \operatorname{cosec}^{-1}\left[\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right]^{1/2}$ **07**

show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + \tan^2 u)$. Also state Euler's modified theorem.

Q.4 **(a)** Evaluate $\lim_{x \to 0}\left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$ **03**

(b) Expand $e^x \sin y$ in powers of x and y upto third degree. **04**

(c) A rectangular box, open at the top, is to have a volume 32 c.c. Find the dimensions of the box requiring least material for its construction. **07**

Q.5 **(a)** Find the volume of the tetrahedron bounded by the plane $x + y + z = 2$ and the planes $x = 0$, $y = 0$ and $z = 0$. **03**

(b) Trace the curve $r = a(1 + \cos\theta)$ **04**

(c) Evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dydx$ by changing the order of integration. **07**

Q.6 **(a)** Evaluate $\int_0^{\infty}\frac{dx}{x^2+1}$ **03**

(b) Evaluate the integral $\int_0^1\int_0^{1-x} e^{\frac{y}{x+y}}\, dydx$ by changing the variables $x + y = u$, $y = uv$ **04**

(c) Evaluate $\iint_R x^2 \, dA$, where R is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x$, $y = 0$ and $x = 8$. **07**

Q.7 **(a)** Test the convergence of $\sum_{n=1}^{\infty} \frac{2\tan^{-1} n}{1+n^2}$ **03**

(b) Find the equation of tangent plane and normal line to the surface $xyz = 6$ at $(1,2,3)$ **04**

(c) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz\,dy\,dx$ **07**

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