Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E.Sem-I/II Examination June-July 2011

Subject code: 110008 **Subject Name: Mathematics-I**

Date: 18/06/11 **Total Marks: 70** **Time: 10:30am to 1:30pm**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as directed. **14**

(a) Show by the definition of limit, $\lim_{x\to 1}(5x-3)=2$

(b) For what value of k is the function

$$f(x)=\begin{cases} kx^2; x\le 2 \\ 3; x>2 \end{cases}$$ Continuous at x = 2 ?

(c) If for the function f, given by

$f(x)=kx^2+7x-4, f'(5)=97$ find the value of k.

(d) Using Rolle's theorem, find points on the curve

$y=16-x^2, x\in[-1,1]$; Where tangent is parallel to x-axis.

(e) Using Lagrange's mean value theorem, show that

$\sin x < x$ for x>0.

(f) Use appropriate mean value theorem to prove

$\frac{\sin b-\sin a}{e^b-e^a}=\frac{\cos c}{e^c}$, for $a<c<b$ and hence deduce that

$e^c\sin x=\left(e^x-1\right)\cos c$

(g) If $\sqrt{5-2x^2}\le f(x)\le\sqrt{5-x^2}$ for $-1\le x\le 1$,

find $\lim_{x\to 0} f(x)$ using Sandwich theorem.

Q.2 **(a)** (i) Expand $\log x$ in power of (x – 1) by Taylor's theorem and hence find the value of $\log_e 1.1$ **04**

(ii) Find the maximum and minimum values of $f(x)=x+\sin 2x$ in the interval $[0,2\pi]$ **03**

(b) (i) State fundamental theorem for definite integral and using it, find the average value of $f(x)=3-\frac{3}{2}x$ on $[0,2]$ and where f actually takes on this value at some point in the given domain. **04**

(ii) Evaluate the improper integral $\int_0^\infty \frac{1}{x^2}dx$ **03**

OR

(b) (i) Test the convergence for following series **04**

(1) $1+\frac{x}{2}+\frac{x^2}{3^2}+\frac{x^3}{4^3}+----\infty, x>0$

(2) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$

(ii) Determine whether $\int_{\infty}^{-\infty}\frac{1}{1+x^2}dx$ converges ? **03**

Q.3 **(a)** Attempt the following. **05**

(i) Discuss the continuity of the given function

$$f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2}; x\neq 0, y\neq 0\\ 0; x=0, y=0\end{cases}$$

(ii) If $u=\log(\tan x+\tan y)$ then prove that $\sin 2x\dfrac{\partial u}{\partial x}+\sin 2y\dfrac{\partial u}{\partial y}=2$

(b) State and prove Euler's theorem on homogeneous function of two variables and apply it to evaluate **05**

$x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}$ for $u=\dfrac{x^3y^3}{x^3+y^3}$

(c) If $x=\sqrt{vw}, y=\sqrt{wu}, z=\sqrt{uv}$ and $u=r\sin\theta\cos\phi, v=r\sin\theta\sin\phi$ **04**

$w=r\cos\theta$. Evaluate $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

OR

Q.3 **(a)** Attempt the following. **05**

(i) Find $\dfrac{dy}{dx}$ when $y^{x^y}=\sin x$

(ii) If $u=u(y-z,z-x,x-y)$ then prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$

(b) Using Lagrange's method of undetermined multipliers, find the maximum value of $u=x^p y^q z^r$ when the variable x, y, z are subject to the condition $ax+by+cz=p+q+r$ **05**

(c) Examine $f(x,y)=x^3+y^3-3axy$ for maximum and minimum values. **04**

Q.4 **(a)** Evaluate $\iint (x+y)^2\,dxdy$ over the curve bounded by the ellipse **05**

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

(b) Sketch the region of integration, change the order of integration and evaluate the integral $I=\int_0^1\int_{x^2}^{2-x} xy\,dxdy$ **05**

(c) Find the area lying inside the cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$, by double integration. **04**

OR

Q.4 **(a)** Evaluate the integral $\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}\,dxdydz$ **05**

(b) Using the transformation $x+y=u, y=uv$, show that **05**

$\iint [xy(1-x-y)]^{\frac{1}{2}}dxdy=\dfrac{2\pi}{105}$, integration being taken over the area of the triangle bounded by the lines $x=0, y=0, x+y=1$

(c) Find the volume of the cylinder $x^2+y^2-ax=0$ bounded by the planes z = 0 and z = x. **04**

Q.5	**(a)**	Determine the constants a and b so that the surface $5x^2 - 2yz - 9x = 0$ be orthogonal to the surface $ax^2y + bz^3 = 4$ at the point (1,-1,2)	**05**
	(b)	Determine $f(r)$, so that the vector $f(r)\overline{r}$ is both Solenoidal and Irrotational.	**05**
	(c)	If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$. Show that $\nabla u, \nabla v, \nabla w$ are coplanar.	**04**

OR

Q.5	**(a)**	State Green's theorem and using it, evaluate $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$; where c is the boundary of the region Bounded by $x \geq 0, y \leq 0$ and $2x - 3y = 6$	**05**
	(b)	Apply Stoke's theorem to find the value of $\int_C (ydx + zdy + xdz)$; Where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$	**05**
	(c)	Evaluate $\oint_C \frac{-y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy$; where $c = c_1 \cup c_2$ with $c_1 : x^2 + y^2 = 1$ and $c_2 : x = \pm 2, y = \pm 2$	**04**

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