Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2016

Subject Code: 110008 **Date:02/06/2016**

Subject Name:Maths-I

Time: 02:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	(i) State Sandwich theorem and using it find $\lim_{x\to 0} g(x)$ if $3-x^2 \le g(x) \le 3\sec x$ for all x.	**03**
		(ii) Find the point 'c' of Mean Value Theorem for the function f(x) = 1-x^2 in $0 \le x \le 2$.	**04**
	(b)	(i) Evaluate using L' Hospital rule $\lim_{x\to 0} \frac{x^2+2\cos x-2}{x\sin^3 x}$	**03**
		(ii)Find the Taylor's series expansion of f(x) = x^3-2x+4 about a=2	**04**
Q.2	**(a)**	(i) Trace the curve r = a (1+ cosθ) ; a>0	**04**
		(ii) Evaluate improper integral $\int_0^\infty \frac{1}{x^2}\,dx$	**03**
	(b)	(i) Is $\int_4^\infty \frac{\sin^2 x}{\sqrt{x}(x-1)}dx$ convergent?	**04**
		(ii) find the extreme values of $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$	**03**
Q.3	**(a)**	(i) Check that the sequence $a_n=\frac{n}{n^2+1}$ is decreasing and bounded below. Is it convergent?	**04**
		(ii) Test the convergence $\sum_{n=1}^{\infty}\frac{n^3+2}{2^n+2}$	**03**
	(b)	Test the convergence of following series:- (i) $\sum_{n=1}^{\infty} ne^{-n^2}$ (ii) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\log(n+1)}$	**07**
Q.4	**(a)**	(i) If $u=\sin^{-1}\left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}\right)$, prove that x$\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{20}\tan u$	**04**
		(ii) If u= x^2-y^2, where $x=2r-3s+4$, y = -r + 8s -5 , find $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial s}$	**03**
	(b)	(i) If u = x + y and v = $\frac{x}{x+y}$ find $\frac{\partial(u,v)}{\partial(x,y)}$	**03**
		(ii) Find the equation of normal line and tangent line at the point (1,1,1) on the surface $x^2+y^2+z^2=3$.	**04**
Q.5	**(a)**	(i) Find the area included between the cardioids r = a(1+cosθ) and r = a(1-cosθ).	**04**
		(ii) Evaluate: $\int_0^4\int_0^{x^2} e^{\frac{y}{x}}\,dydx$	**03**
	(b)	(i) Use Lagrange method of undetermined multipliers to find the shortest distance from the point (1,2,2) to the sphere $x^2+y^2+z^2=16$	**07**

Q.6	**(a)**	(i) Evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dA$ by changing the order of integration.	**04**
		(ii) Find the volume of the solid of revolution of the area bounded by the curve $y = xe^x$ and the straight lines x=1 , y=0.	**03**
	(b)	(i) Evaluate: $\int_0^1 \int_0^{\sqrt{z}} \int_0^{2\pi} (r^2 cos\theta^2 + z^2)\, r\, d\theta dr dz$	**03**
		(ii) Find the directional derivative of $f(x, y, z) = xyz$ at the point (-1,1,3) in the direction of the vector $\overline{a} = \overline{\imath} - 2\overline{\jmath} + 2\overline{k}$.	**04**
Q.7	**(a)**	(i) Evaluate $\int_0^4 x^3\sqrt{4x - x^2}\, dx$	**03**
		(ii) Find grad$(\emptyset) = \log(x^2 + y^2 + z^2)$ at the point (1,0,-2)	**04**
	(b)	(i) Using Green's Theorem, evaluate the line integral $\int_c (siny\, dx + cosx\, dy)$ counter clockwise, where C is the boundary of the triangle with vertices (0,0) and (π,0)	**04**
		(ii) Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{\imath}+(3xz+2xy)\vec{\jmath} + (3xy - 2xz + 2z)\vec{k}$ is irrotational.	**03**

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