Seat No.: _____ Enrolment No.______

GUJARAT TECHNOLOGICAL UNIVERSITY

B.E. Sem-II Remedial Examination September 2009

Subject code: 110009
Subject Name:Maths-II

Date: 09/09/2009 **Time: 11:00am-02:00pm**

Total Marks: 70

Instructions:

1. **Write seat no. and enrolment no. at given location on question paper.**
2. **Attempt all questions.**
3. **Make suitable assumptions wherever necessary.**
4. **Figures to the right indicate full marks.**

Q.1			
		Attempt the following(**each carry equal marks**).	**14**
	(a)	Find two vectors in R^2 with Euclidean norm 1 whose Euclidean	
		inner product with (3, -1) is zero.	
	(b)	If $\bar{u}$ and $\bar{v}$ are orthogonal vectors in an inner product space	
		then prove that $\|\bar{u}+\bar{v}\|^2=\|\bar{u}\|^2+\|\bar{v}\|^2$	
	(c)	Show that $f_1(x)=1, f_2(x)=e^x and\ f_3(x)=e^{2x}$ form a linearly	
		independent set of vectors in $C^2(-\infty,\infty)$.	
	(d)	If $\bar{v}_1=(4,6,8)$, $\bar{v}_2=(2,3,4)$, $\bar{v}_3=(-2,-3,-4)$ are three vectors in R^3	
		that have initial points at the origin. Are they lie on the same	
		line ?	
	(e)	Using caley – Hamilton theorem , find A^2 ,if $A=\begin{bmatrix}1&4\\2&3\end{bmatrix}$.	
	(f)	Express the matrix A as the sum of a symmetric and skew -	
		Symmetric matrix, where A = $\begin{bmatrix}4&2&-3\\1&3&-6\\-5&0&7\end{bmatrix}$	
	(g)	Show that the characteristic equation of a 2×2 matrix A can be	
		expressed as $\lambda^2-tr(A)\lambda+\det(A)=0$,	
		where tr(A) is the trace of A.	
Q.2			
	(a)	Obtain the reduced row echelon form of the matrix	**04**
		A = $\begin{bmatrix}1&3&2&2\\1&2&1&3\\2&4&3&4\\3&7&4&8\end{bmatrix}$	
		and hence find the rank of the matrix A.	
	(b)	Find the eigenvalues and eigenvectors of the matrix A and	**03**
		discuss about its algebraic and Geometric multiplicity ,where	
		A = $\begin{bmatrix}0&1&1\\1&0&1\\1&1&0\end{bmatrix}$	

	(c)	Attempt the following:	
	(i)	Solve:	**04**
		$x+y+2z=9$ $2x+4y-3z=1$ by Gaussian elimination and back substitution. $3x+6y-5z=0$	
	(ii)	Show that A = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is orthogonal.	**03**
		OR	
	(c)	Attempt the following:	
	(i)	Find inverse of the matrix A by Gauss-Jordan method , Where	**04**
		A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.	
	(ii)	Prove that the matrix A = $\begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$ is a Hermitian	**03**
		and iA is a skew Hermitian matrix.	
Q.3			
	(a)	The set V of all pairs of real numbers of the form $(1,x)$ with	**05**
		the operations defined as $(1,y)+(1,y')=(1,y+y')$ and	
		$k(1,y)=(1,ky)$ is a vector space.	
	(b)	Is the following set of vectors in P_2 linearly independent ?	**05**
		$2-x+4x^2, 3+6x+2x^2, 2+10x-4x^2$.	
	(c)	Show that the $\overline{w}=(9,2,7)$ is a linear combination of the vectors	**04**
		$\overline{u}=(1,2,-1)$ and $\overline{v}=(6,4,2)$ in R^3.	
		OR	
Q.3			
	(a)	Find bases for the row space and column space of	**05**
		A = $\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$.	
	(b)	Let S = $\{\overline{v}_1, \overline{v}_2, \overline{v}_3\}$ be the basis for R^3 , Where	**05**
		$\overline{v}_1=(1,2,1), \overline{v}_2=(2,9,0), \overline{v}_3=(3,3,4)$	
		(i) Find the coordinate vector of $\overline{v}$ = (5 , -1 ,9) with respect to S.	
		(ii) Find the vector $\overline{v}$ in R^3 whose coordinate vector with respect	
		to the basis S is $(\overline{v})_s$ = (-1 ,3 ,2).	
	(c)	Check whether the following are subspace of R^3.	**04**
		Justify your answer.	
		(i) W = $\{(a,0,0)/a\in R\}$	
		(ii) w = $\{(x,y,z)/x^2+y^2+z^2\le 1\}$.	

Q.4			
	(a)	Find the rank and nullity of the matrix A = $\begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$.	**05**
	(b)	Let S = $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ be the basis for R^3 , Where	**05**
		$\bar{v}_1 = (1,1,1),\ \bar{v}_2 = (1,1,0),\ \bar{v}_3 = (1,0,0)$ .Let T: $R^3 \to R^3$ be the linear	
		operator such that $T(\bar{v}_1) = (2,-1,4), T(\bar{v}_2) = (3,0,1), T(\bar{v}_3) = (-1,5,1)$.	
		Find a formula for T (x_1, x_2, x_3) and then compute T (2, 4 ,-1).	
	(c)	Determine whether the function is a linear transformation.	**04**
		Justify your answer.	
		(i) T: $M_{nn} \to R$, where T(A) = tr(A).	
		(ii)) T: $M_{nn} \to R$, where T(A)=det(A).	
		OR	
Q. 4			
	(a)	T: $R^2 \to R^3$ be the linear transformation defined by	**05**
		$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$.	
		Find the matrix for the transformation T with respect to the bases	
		$B = \{\bar{u}_1, \bar{u}_2\}$ for R^2 and $B' = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ for R^3, where	
		$\bar{u}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \bar{u}_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix} : \bar{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \bar{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \bar{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.	
	(b)	T: $R^2 \to R^2$ is defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \left(\begin{bmatrix} x_1 - 2x_2 \\ -x_2 \end{bmatrix}\right)$	**05**
		B= $\{\bar{u}_1, \bar{u}_2\}$ and $B' = \{\bar{v}_1, \bar{v}_2\}$,where	
		$\bar{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \bar{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} : \bar{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \bar{v}_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$	
		(i) Find the matrix of T with respect to the basis B.	
		(ii) Find the transition matrix P from B' to B.	
		(iii) Find matrix of T with respect to the basis B' .	
	(c)	Attempt the following:	**04**
		(i) Use matrix multiplication to find the reflection of (2, -5,3)	
		about the x y plane.	
		(i) Use matrix multiplication to find the image of the vector	
		(-2 , 1,2) if it is rotated 45^0 about the Y- axis.	
Q.5			
	(a)	Find a matrix P that diagonalizes A, and determine $P^{-1}AP$,	**05**
		Where A = $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$.	
	(b)	Let R^3 have the Euclidean inner product.Use Gram –Schmidt	**05**
		Process to transform the basis $\{\bar{u}_1, \bar{u}_2, \bar{u}_3\}$ into an orthonormal basis	
		where $\bar{u}_1 = (1,1,1), \bar{u}_2 = (-1,1,0), \bar{u}_3 = (1,2,1)$.	

	(c)	Attempt the following:	**04**
		(i) Show that $\langle \bar{u}, \bar{v} \rangle = 9u_1v_1 + 4u_2v_2$ is the inner product on R^2	
		Generated by A = $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$	
		(ii) Let R^2 have the Euclidean inner product . Find the cosine of the	
		angle θ between the vectors $\bar{u}$ = (4,3,1,-2) and $\bar{v}$ =(-2,1,2,3).	
		OR	
Q.5			
	(a)	Find the maximum and minimum values of the quadratic form	**05**
		$x_1^2 + x_2^2 + 4x_1x_2$ subject to the constraint $x_1^2 + x_2^2 = 1$,and determine	
		Values of x_1, x_2 at which the maximum and minimum occur.	
	(b)	Find the least square solution of the linear system **Ax = b**	**05**
		given by $x_1 - x_2 = 4, 3x_1 + 2x_2 = 1, -2x_1 + 4x_2 = 3$ and find the	
		Orthogonal projection of **b** on the column space of A.	
	(c)	Attempt the following	**04**
		(i) Let R^3 have the Euclidean inner product. For which values of k	
		are $\bar{u}$ and $\bar{v}$ orthogonal ?	
		(a) $\bar{u}$ = (k , k , 1) , $\bar{v}$ = (k , 5 , 6)	
		(b) $\bar{u}$ = (2 , 1 , 3) , $\bar{v}$ = (1 ,7 , k)	
		(ii) If $\bar{u} = (u_1, u_2)$ and $\bar{v} = (v_1, v_2)$ be vectors in R^2 . Verify that the	
		weighted Euclidean inner product $\langle \bar{u}, \bar{v} \rangle = 3u_1v_1 + 2u_2v_2$ satisfies	
		the four inner product axioms.	

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