Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER 1st / 2nd EXAMINATION (OLD SYLLABUS) – SUMMER - 2017

Subject Code: 110009 **Date:29/05/2017**

Subject Name: Maths-II

Time: 2:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** **(i)** What conditions must b_1, b_2, b_3 satisfy in order for $x_1 + 2x_2 + 3x_3 = b_1, 2x_1 + 5x_2 + 3x_3 = b_2, x_1 + 8x_3 = b_3$ be consistent? **04**

(ii) Obtain the reduced row echelon form of the matrix $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$ **03**

(b) Solve using Gauss-Elimination method **07**

$x + y + 2z = 8\ , -x - 2y + 3z = 1\ , 3x - 7y + 4z = 10$

Q.2 **(a)**

(i) Determine the rank of the matrix A if $A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$ **04**

(ii) Is $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + 3y, y, z + 2x)$ linear? Is it one-to-one, onto or both? Justify. **03**

(b) By Gauss-Jordan method find inverse of $\begin{bmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 6 & 8 \end{bmatrix}$ **07**

Q.3 **(a)** **(i)** Find a basis for the orthogonal complement of the subset of R^3 spanned by the vectors $v_1 = (1, -1, 3), v_2 = (5, -4, -4)$ and $v_3 = (7, -6, 2)$. **04**

(ii) Let $W = \mathrm{Span}\left\{\left(\frac{4}{5}, 0, -\frac{4}{5}\right), (0, 1, 0)\right\}$. Express $w = (1, 2, 3)$ in the form of $w = w_1 + w_2$ where $w_1 \in W$ and $w_2 \in W^{\perp}$. **03**

(b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis vectors $\overline{u_1} = (1,1,1), \overline{u_2} = (-1,1,0)$ and $\overline{u_3} = (1,2,1)$ into orthonormal basis $\{v_1, v_2, v_3\}$ and show that the set $V = \{(1,0,0), (0,1,0), (0,0,1)\}$ is an orthonormal set in R^3 with Euclidean inner product. **07**

Q.4 **(a)**

(i) Express the matrix $A = \begin{bmatrix} 2i & 3i & 4i \\ 0 & 2 & 5i \\ 2+i & 1-i & 0 \end{bmatrix}$ as a sum of Hermitian and Skew-Hermitian matrices. **04**

(ii) Verify that the matrix A is unitary, where $A = \frac{1}{3}\begin{bmatrix} 2+i & 2i \\ 2i & 1-i \end{bmatrix}$ **03**

(b) Consider the basis $S = \{v_1, v_2\}$ for R^2, where $v_1 = (-2,1)$ and $v_2 = (1,3)$ and let $T : R^2 \to R^3$ be linear transformation such that $T(v_1) = (-1,2,0)$ and $T(v_2) = (0,-3,5)$. Find a formula for $T(x_1, x_2)$ and use that formula to find $T(2,-3)$. **07**

Q.5 **(a)** **(i)** Show that the set of vectors $\{2x^3 + x^2 + x + 1, x^3 + 3x^2 + x - 2, x^3 + 2x^2 - x + 3\}$ in P_3 is linearly independent. **04**

(ii) Find basis and dimension of **03**

$$W = \{(a_1, a_2, a_3, a_4) \in R^4 / a_1 + a_2 = 0, a_2 + a_3 = 0, a_3 + a_4 = 0\}$$

(b) Check whether $V = R^2$ is a vector space with respect to the operations $(u_1, u_2) + (v_1, v_2) = (u_1 + v_1 - 2, u_2 + v_2 - 3)$ and $\alpha(u_1, u_2) = (\alpha u_1 + 2\alpha - 2, \alpha u_2 - 3\alpha + 3), \alpha \in R$. **07**

Q.6 **(a)** **(i)** Determine which of the following is subspace or not **04**

(1) $V = \{(x, y) \mid x = 3y\}$ in R^2 (2) $V = \{(x, y) \mid x^2 = y^2\}$ in R^2

(ii) Determine whether the linear transformation is one-to-one or not **03**

$T : R^2 \to R^2$ where $T(x, y) = (x + y, x - y)$

(b) Let $T_1 : R^2 \to R^3, T_2 : R^3 \to R^3, T_3 : R^3 \to R^2$ be the linear transformation given by $T_1(x, y) = (-2y, 3x, x - 2y), T_2(x, y, z) = (y, z, x), T_3(x, y, z) = (x + z, y - z)$ .find the domain and codomain of $(T_1 \circ T_2 \circ T_3)(x, y)$. Also find $(T_3 \circ T_2 \circ T_1)(1,1)$. **07**

Q.7 **(a)**

(i) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. **04**

(ii) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 3 & 2 \\ 4 & -1 \end{bmatrix}$ **03**

(b) Find a matrix P that diagonalizes the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ and hence find A^{13}. **07**