Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2016

Subject Code: 110014 **Date: 02/06/2016**

Subject Name: Calculus

Time:02:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i)Is the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+......$ convergent? Does it converge absolutely? **04**

(ii) Evaluate $\int_{0}^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ **03**

(b) (i) Expand $\sin\left(\frac{\pi}{4}+x\right)$ in power of x. Hence find the value of $\sin 44^0$. **04**

(ii) Evaluate $\lim_{x \longrightarrow \frac{\pi}{2}} (\sec x - \tan x)$ **03**

Q.2 **(a)** (i) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin u \cos 3u$. **04**

(ii) If $u=f(x-y, y-z, z-x)$; show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **03**

(b) (i) Show that $f(x,y)=\begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & ; (x,y)\neq 0 \\ 0 & ; (x,y)\neq 0 \end{cases}$ is continuous at origin. **04**

(ii) Evaluate the improper integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. **03**

Q.3 **(a)** (i) Find the maximum and minimum values and saddle point of $f(x,y)=x^2+y^2+4x+6y+13$. **04**

(ii) Find the equation of plane and normal line at a point $(3,4,5)$ to the surface $x^2+y^2-4z=5$. **03**

(b) (i) Find a point on the plane $2x+3y-z=5$ which is nearest to origin, using Lagrange's method of undermined multipliers. **04**

(ii) Find the sum of the series if it converges $5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+.........$ **03**

Q.4 **(a)** (ii) Expand $f(x)=e^{\sin x}$ by Maclurin's series. **04**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{2n^2+3n}{5+n^5}$. **03**

(b) (i) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$. **04**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n^3+2}{2^n+2}$. **03**

Q.5 **(a)** (i) Evaluate $\iint_R xy(x+y)dA$ over the area between $y=x^2$ *and* $y=x$. **04**

(ii) Evaluate $\iint_R (x^2+y^2)xdxdy$ over the positive quadrant of the circle $x^2+y^2=a^2$ by polar coordinates. **03**

(b) (i) Change the order of integration $\int_0^a\int_y^a \frac{x}{x^2+y^2}dxdy$. **04**

(ii) Evaluate $\int_0^1\int_0^y\int_0^{x+2y}(x+y+z)dzdxdy$. **03**

Q.6 **(a)** (i) Trace the curve $r=a(1+\cos\theta)$. **04**

(ii) Use the fundamental theorem to find $\frac{dy}{dx}$ if $y=\int_1^{x^2}\cos t\ dt$. **03**

(b) (i) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **04**

(ii) Find the volume of the region between the curve $y=\sqrt{x}$, $0\le x\le 4$ and the line $x=4$ revolved about the $x-$axis to generate a solid. **03**

Q.7 **(a)** (i) Evaluate $\iint_R (x^2-y^2)^2 dA$ over the area bounded by the lines $|x|+|y|=1$ using transformations $x+y=u$; $x-y=v$. **04**

(ii) Evaluate $\lim_{x\to 0}\left(\frac{e^x+e^{2x}+e^{3x}}{3}\right)^{\frac{1}{x}}$ **03**

(b) (i) Find the area of the region R enclosed by the parabola $y=x^2$ and the line $y=x+2$. **04**

(ii) If $x^3+y^3-3axy=0$ find $\frac{dy}{dx}$. **03**

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