Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st /2nd (OLD SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code:110015 **Date: 15/06/2015**

Subject Name: Vector Calculus and Linear Algebra

Time: 10.30am-01.30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** State Caley-Hamilton theorem and hence find inverse of **07**

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

(b) Find the Eigen values of A^{-1}, A^{10}, $A + 3I$ where **07**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 1 & -1 & 3 \end{bmatrix}$$. Also find det(A).

Q.2 **(a)** Find rank and nullity of the matrix **07**

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

and state rank-nullity theorem.

(b) Find bases for the row and column space of **07**

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

Q.3 **(a)** (1) Check $W = \{(x, y) / xy \geq 0\}$ is a subspace of R^2? Justify. **02**

(2) Check $W = \{(x, y, z) / x^2 + y^2 + z^2 = 1\}$ is a subspace of R^3? Justify. **02**

(3) Define: Basis, Subspace, skew symmetric matrix **03**

(b) (1) Let R^4 have the Euclidean inner product. Fine the cosine of the angle θ between the vectors $u = (4,3,1,-2)$ and $v = (-2,1,2,3)$ **02**

(2) If u and v are orthogonal vectors in an inner product space, then prove that $\|u + v\|^2 = \|u\|^2 + \|v\|^2$ **02**

(3) Solve by Cramer's rule **03**

$$x + y + 2z = 8$$
$$-x - 2y + 3z = 1$$
$$3x - 7y + 4z = 10$$

Q.4 **(a)** Solve by Gauss-Jordan method **07**

$$\begin{aligned} x_1 + 3x_2 - 2x_3 \quad + 2x_5 \quad &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 + 10x_4 \quad + 15x_6 &= 5 \\ 2x_1 + 6x_2 \quad + 8x_4 + 4x_5 + 18x_6 &= 6 \end{aligned}$$

(b) Consider the vector space R^3 with the Euclidean inner product. $u_1 = (1,1,1)$, $u_2 = (0,1,1)$, $u_3 = (0,0,1)$. Apply Gram-Schmidt process to find and orthogonal basis. Also normalize the orthogonal basis into orthonormal basis. **07**

Q.5 **(a)** (1) Let $T: M_{nn} \to R$ be the transformation that maps an $n \times n$ $(n > 1)$ matrix into its determinant: $T(A) = \det(A)$ whether T is a linear transformation? **03**

(2) Let $T_A : R^4 \to R^4$ be multiplication by **04**

$$A = \begin{bmatrix} 1 & 3 & -2 & 4 \\ 2 & 6 & -4 & 8 \\ 3 & 9 & 1 & 5 \\ 1 & 1 & 4 & 8 \end{bmatrix}$$

Determine whether T_A is one to one. State the result which you have used.

(b) Find algebraic and geometric multiplicity of A, where **07**

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Q.6 **(a)** Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $x = 0$, $y = 0$, $x + y = 1$ **07**

(b) Use Stoke's theorem to calculate the circulation of the field $F = (y^2 + z^2)i + (x^2 + z^2)j + (y^2 + x^2)k$ around the curve C:The boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above. **07**

Q.7 **(a)** Suppose that the set V is the set of positive real numbers with addition and scalar multiplication defined as follows **07**

$$x + y = xy$$

$$ax = x^a$$

Prove that V is a vector space.

(b) Show that the quadratic $5x^2 + 6y^2 + 7z^2 - 4xy + 4yz = 162$ is an ellipsoid. **07**

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